Unit 06

CIRCLE

Q.1 Define following terms.

(i) Circle

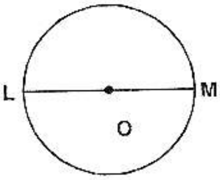
A circle is the set of all points in a plane which are equidistant from a fixed point of the plane. This fixed point is called centre of circle and is not included in the points of circle.

(ii) Circumference of circle

The length of the line joining all points of the circle is called circumference of the circle. Its formula is $c = 2\pi r$.

(iii) Diameter of a circle

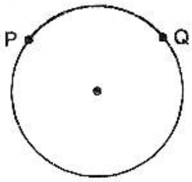
A chord passing through the centre of the circle is called a diameter of the circle.



In figure, $\overline{LM}$ is a diameter of the circle.

(iv) Arc of circle

Any portion or part of a circle is called arc of the circle.



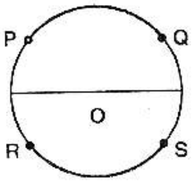
In figure, part PQ is an arc of the circle.

(v) Minor arc

An arc which is included in a semi circle is called minor arc.

(vi) Major arc

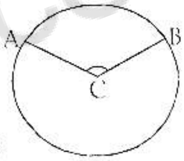
An arc which includes a semi circle is called major arc.



In figure $\widehat{PQ}$ is a minor arc and $\widehat{RPS}$ is a major arc.

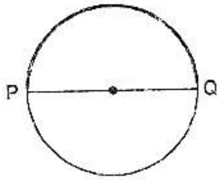
(vii) Central angle

The angle subtended by an arc at the centre of circle is called a central angle. In fig. $\angle ACB$ is central angle.



(viii) Half circle or semi-circle

The portion of a circle intercepted by any central chord is a half circle, or semi-circle.



(ix) Congruent circles

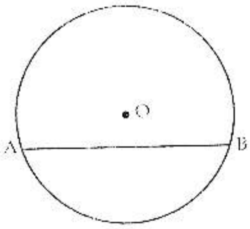
Two circles are congruent if their radii are equal.

(x) Congruent arcs

Two arcs of the same circle or of different circles of same radii are called congruent arcs if their lengths are equal.

(xi) Chord of circle

A line segment whose end points are any two points of a circle is called a

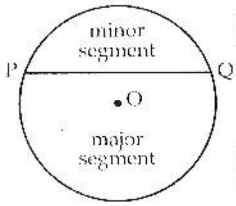


chord of the circle.

If fig. $\overline{AB}$ is the chord of circle.

(xii) Segment of a circle

A chord of a circle divides the circular region into two parts. These parts are called segments of the circle.



(xiii) Minor segment

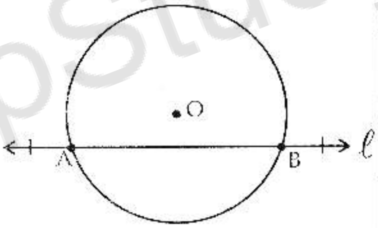
The region bounded by the chord and the minor arc is called minor segment.

(xiv) Major segment

The region bounded by the chord and the major arc is called major segment.

(xv) Secant of a circle

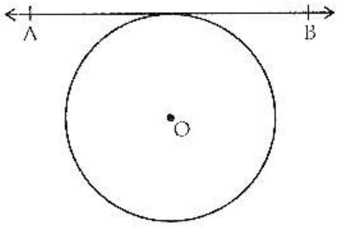
A line which has two distinct points in common with a circle is called a secant line.



In fig. Line ℓ is a secant line of circle.

(xvi) Tangent line

A line which touches a circle at one point only and none of its points lie in the interior of the circle is called a tangent line to the circle.

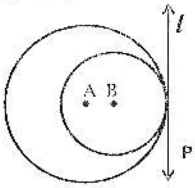


In fig. $\overleftrightarrow{AB}$ is a tangent to the circle.

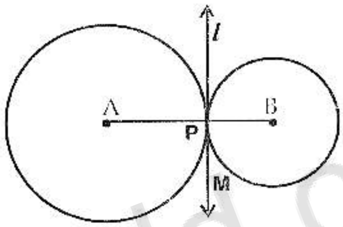
(xvii) Tangent circles

Those circles which have only one point in common are called tangent circles. Two circles can have one point in common in two ways:-

(i) one circle is inside the another circle.



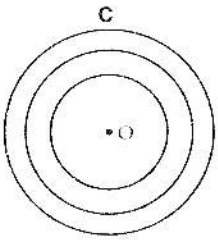
(ii)one circle is outside the other circle.



The tangent line at their common point is called their common tangent. The tangent at the common point of the two circles is called a common tangent to the two circles.

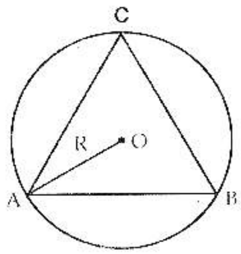
(xviii) Concentric circle

Circles having a common centre are called concentric circles as in fig.



(xix) Circum circle of a triangle

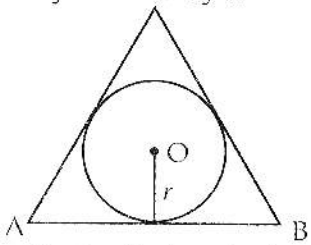
The circle which passes through the three vertices of a triangle is called circum circle of the triangle. Its centre is called circum centre and radius is called circum radius, usually denoted by R.



(xx) Inscribed circle of a triangle

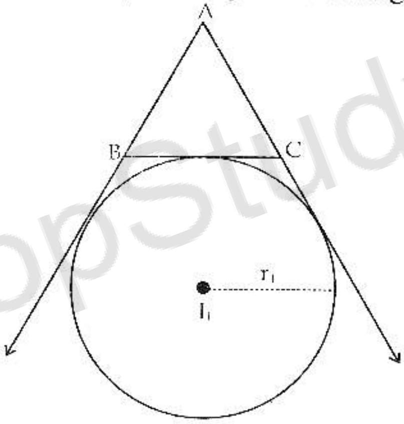
A circle touching the three sides of a triangle internally is called inscribed circle of the triangle. Its centre is

called in-centre and radius is called in-radius, usually denoted by r.



(xxi) Escribed circle (e-circle) of a triangle

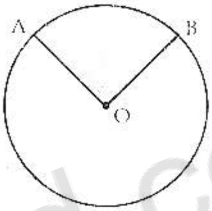
The circle which touches one side externally and the other two produced sides of a triangle internally is called an escribed circle (e-circle) of the triangle.



(xxii) Sector of a circle

The circular region bounded by an arc of a circle and its two corresponding radial segments is called a sector of circle.

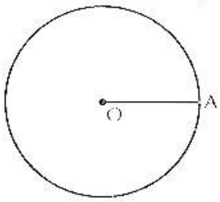
In fig. shaded region is the sector of circle.



(xxiii) Radial segment

A line segment joining the centre of circle to any point on the circle is called radial segment. All radial segment of a circle are equal in length. The length of a radial segment is called radius.

In fig. $\overline{OA}$ is a radial segment.



THEOREMS

Theorem 1

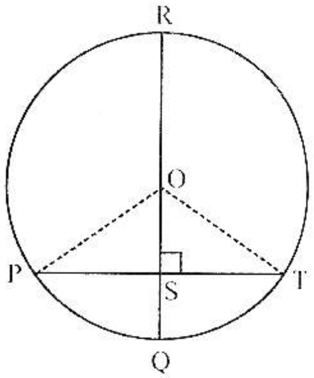
If a diameter of a circle is perpendicular to a chord, it bisects the chord.

Given:

$\overline{PQ}$ is a diameter of the circle with centre O. The diameter $\overline{PQ}$ is perpendicular to the chord $\overline{PT}$ at S.

To Prove: $\overline{PS} \cong \overline{ST}$

Construction: Join O to P and T

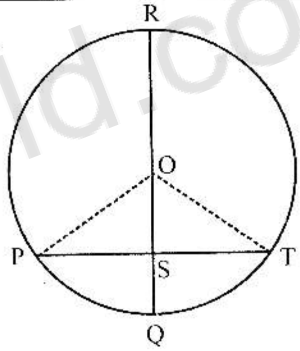


Proof:

Statements	Reasons
In $\Delta OPS \leftrightarrow \Delta OTS$	
$\angle OST \cong \angle OST$	Each is right angle (given)
$\overline{OP} \cong \overline{OT}$	Radii of the same circle
and $\overline{OS} \cong \overline{OS}$	Common
$\therefore$ $\Delta OPS \cong \Delta OTS$	H.S $\cong$ H.S
$\therefore$ $\overline{PS} \cong \overline{ST}$	Corresponding sides of congruent triangles.
Hence $\overline{RQ}$ bisects the chord $\overline{PT}$	

Theorem 1a (Converse of Theorem 1)

If a diameter of a circle bisects a chord it will be perpendicular to the chord.



Given: $\overline{PQ}$ is a diameter of the circle with at O and bisects the chord $\overline{PT}$ at S so that $\overline{PS} \cong \overline{ST}$

To Prove: $m\angle OST = m\angle OSP = 90^{o}$

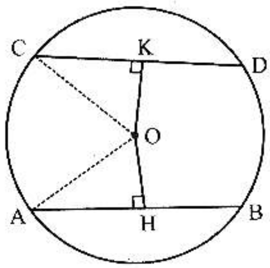
Construction: Join O to P and T

Proof:

Statements	Reasons
In $\Delta OPS \leftrightarrow \Delta OTS$	
$\overline{OP} \cong \overline{OT}$	Radii of the same circle
$\overline{OS} \cong \overline{OS}$	Common side
$\overline{SP} \cong \overline{ST}$	Given
$\therefore$ $\Delta OPS \cong \Delta OTS$	S.S.S $\cong$ S.S.S
$\therefore$ $\angle OPS \cong \angle OST$	Corresponding angles of congruent triangles.
$\therefore$ $m\angle OST = m\angle OSP = 90^{o}$	Adjacent supplementary equal angles, each is right angle.

Note 1:

The perpendicular bisector of a chord of a circle passes through the centre of the circle



Note 2:

The diameter of a circle passes through the mid points of two parallel chords of a circle

Theorem 2

If two chords of a circle are congruent, then they will be equidistant from the centre.

Given:

A circle with centre O and chord $\overline{AB} \cong$ chord $\overline{CD}$

To Prove: $\overline{AB}$ and $\overline{CD}$ are equidistant from the centre O i.e. $\overline{OH} \cong \overline{OK}$

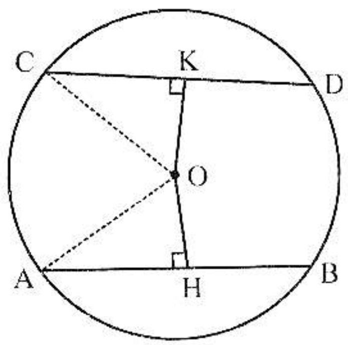
Construction:

Proof: Join O to A and C and draw perpendicular $\overline{OH}$ and $\overline{OK}$ on chord $\overline{AB}$ and $\overline{CD}$ respectively.

Statements	Reasons
$m\overline{AH} = \frac{1}{2}\, m\overline{AB}$	$\overline{OH}$ is perpendicular to $\overline{AB}$ from centre O
and $m\overline{CK} = \frac{1}{2}\, m\overline{CD}$	$\overline{OK}$ is perpendicularly to $\overline{CD}$ form centre O
But $\overline{AB} \cong \overline{CD}$	Given
$\therefore$ $\overline{AH} = \overline{CK}$	Both are $\frac{1}{2}$ of equal segments
Now in	
$\Delta OHA \leftrightarrow \Delta OKC$	
$\overline{OA} \cong \overline{OC}$	radii of the same circle
$m\angle OHA = m\angle OKC$	construction each is 90° proved
$\overline{AH} \cong \overline{CK}$	Proved
$\therefore$ $\Delta OHA \cong \Delta OKC$	H.S $\cong$ H.S
Hence $\overline{OH} \cong \overline{OK}$	Corresponding sides of congruent triangles.

Theorem 2 (Converse of Theorem 2)

Two chords of a circle which are equidistant from the centre, are congruent.



Given:

A circle with centre O and two chords $\overline{AB}$ and $\overline{CD}$ such that perpendiculars $\overline{OH}$, $\overline{OK}$ from O to $\overline{AB}$, $\overline{CD}$ are equal i.e. $\overline{OH} \cong \overline{OK}$

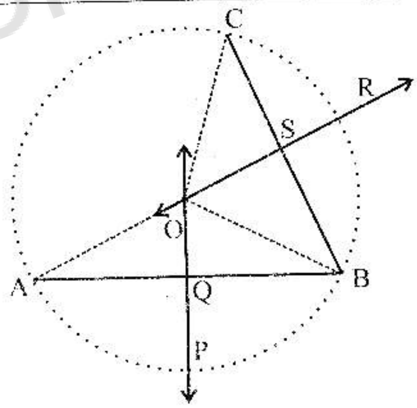
To Prove: $\overline{AB} \cong \overline{CD}$

Construction: Join O to A and C

Proof:

Statements	Reasons
In $\Delta OHA \leftrightarrow \Delta OKC$	
$\overline{OH} \cong \overline{OK}$	Given
$m\angle OHA = m\angle OKC = 90^\circ$	Given
$\overline{OA} \cong \overline{OC}$	Radii of the same circle
$\therefore \Delta OHA \cong \Delta OKC$	H.S $\cong$ H.S
$\therefore \overline{AH} \cong \overline{CK}$	Corresponding sides of congruent triangle
But H and K are midpoints of $\overline{AB}$ and $\overline{CD}$ respectively.	
$\therefore 2m\overline{AH} = 2m\overline{CK}$	Perpendicular from centre of a circle Bisects the chord.
or $\overline{AB} \cong \overline{CD}$	

Theorem 3



One and only one, circle can pass through three non collinear points.

Given: Three points A, B, C not lying on the same line.

To Prove: One and only one, circle can pass through the points A, B and C.

Construction: Join A to B and B to C. Draw the perpendicular bisectors $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$ of $\overline{AB}$ and $\overline{BC}$ respectively to meet each other at O.

Proof:

Statements	Reasons
Since points A,B and C are not lying on the same straight line the perpendicular bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ are not parallel and therefore intersect in point O.	Two lines which are not parallel always intersect at a point.
Now $\overline{OA} \cong \overline{OB}$ (i)	Point O lies on perpendicular bisector of $\overline{AB}$, construction
Similarly, $\overline{OB} \cong \overline{OC}$ (ii)	Point O lies on perpendicular bisector of $\overline{BC}$
$\therefore \overline{OA} \cong \overline{OB} \cong \overline{OC}$	By (i) and (ii) (Axiom)

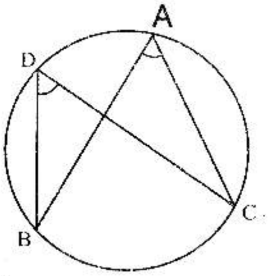
$\therefore$ O is equidistant from point A, B and C, that points A, B and C lie on the circle with centre O. Moreover, as two lines can intersect only in one point so 'O' is the only centre of circle. Hence one and only one circle can pass through three non-collinear points	Definition of a circle.

Note 1: If more than two point lie on the same straight line, then circle can not pass through them.

Note 2: Two circles can not intersect each other at more than two points.

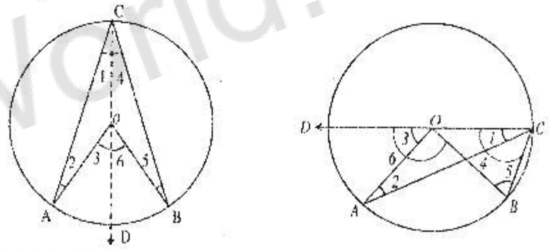
Inscribed angle:

An inscribed angle of an arc is the angle whose vertex is any point of the arc and whose arms pass through the ends of the arc. In the adjoining figure, $\angle BAC$ and $\angle BDC$ are inscribed angles of $\widehat{BC}$.



Theorem 4

The measure of a central angle of a minor arc of a circle, is double in measure of the inscribed angle of the corresponding major arc.



Given:

$\widehat{AB}$ is a minor arc of a circle with centre O. $\angle ACB$ is the angle inscribed in the corresponding major arc and $\angle AOB$ is the central angle of minor $\widehat{AB}$.

To Prove:

$$m\angle AOB = 2m\angle ACB$$

Construction:

Extend $\overrightarrow{CO}$ to meet the circle again in D, $\overline{CD}$ is a diameter of the circle.

Proof:

Statements	Reasons
In ΔAOC	
$m\overline{OA} = m\overline{OC}$	Radii of same circle
$m\angle 1 = m\angle 2$	Measures of angle opposite to congruent sides.
	Sum of non adjacent interior angles

Now $m\angle 3 = m\angle 1 + m\angle 2$

$\therefore\ m\angle 3 = m\angle 1 + m\angle 1$

$m\angle 3 = 2\,m\angle 1$ (I)

Similarly from ΔBOC

$m\angle 6 = 2\,m\angle BCO$

$m\angle 6 = 2\,m\angle 4$ (II)

In figure (i),

$m\angle 3 + m\angle 6 = 2\,(m\angle 1 + m\angle 4)$

i.e. $m\angle AOB = 2m\angle ACB$

Similarly, by subtracting (I) and (II) for figure (ii), we get the required result.

equal to opposite exterior angle.

$m\angle 2 = m\angle 1$ (Proved)

Adding (I) and (II)

Note-1:

All angles inscribed in the same arc are equal in measure. In the figure, C is the centre. *ACB* is the central angle made by the minor arc *AB* ....(i)

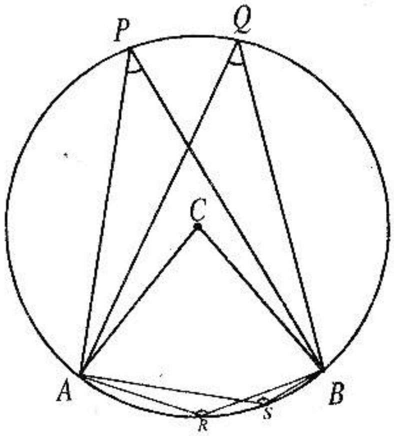
$m\angle ACB = 2m\angle APB$ (i) Proved

$m\angle ACB = 2m\angle AQB$ (ii) Proved

$2m\angle APB = 2m\angle AQB$

From (i) and (ii)

$\therefore\ m\angle APB = m\angle AQB$



Note-2:

Angle inscribed in a major arc is acute and those in minor arc obtuse.

This follows from the fact that minor arc AB subtends a central angle less than 180^o and major arc subtends more than 180^o.

So $2m\angle APB = m\angle ACB$ (i)

But $m\angle ACB < 180^o$ (ii)

From (i) and (ii), we get

$2\,m\angle APB < 180^o$

or $m\angle APB < 90^o$

i.e $m\angle APB$ is acute.

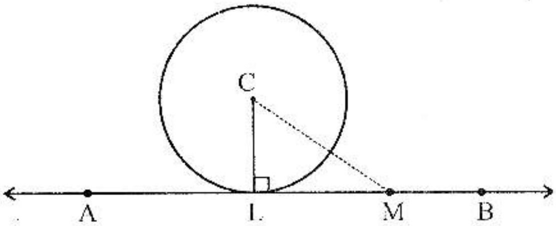
Similarly, $m\angle ARB$ and $m\angle ASB$ are obtuse.

Note-3:

In congruent circles or in the same circle, if two minor arcs are congruent, then the angles inscribed by their corresponding major arcs are also congruent.

Theorem 5

A line which is perpendicular to a radial segment of a circle at its outer end on the circle, is a tangent to the circle.



Given:

A circle with centre C. $\overline{CL}$ is its radial segment. $\overleftrightarrow{AB}$ is perpendicular to $\overline{CL}$ at its outer end L.

To Prove:

$\overleftrightarrow{AB}$ is a tangent to the circle, i.e. $\overleftrightarrow{AB}$ touches the circle at L.

Construction:

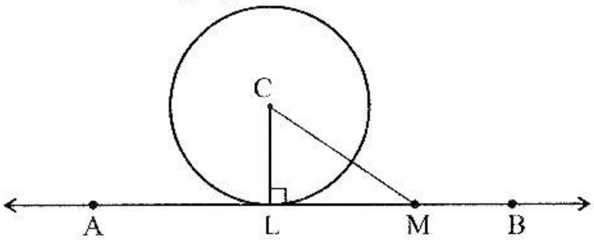
Take any point M on $\overleftrightarrow{AB}$. Join C to M

Proof:

Statements	Reasons
In ΔCLM,	
$m\angle CLM = 90^\circ$	$\because \overline{CL} \perp \overleftrightarrow{AB}$ (given)
and $m\angle CML < 90^\circ$	acute angle of right angled triangle.
$m\,\overline{CM} > m\,\overline{CL}$	Greater angle has greater side opposite to it.
but $\overline{CL}$ is equal to the radius of the circle.	
$\therefore$ M is a point outside the circle	
Similarly, every point on $\overleftrightarrow{AB}$ except L lies outside the circle.	
Hence $\overleftrightarrow{AB}$ intersects the circle at one point only.	
$\therefore$ $\overleftrightarrow{AB}$ is a tangent to the circle at one point only.	

Theorem 5a **(Converse of theorem 5)**

The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other .



Given:

Given a circle with centre C. $\overleftrightarrow{AB}$ is tangent to the circle at point L.

To Prove:

Line AB and radial segment $\overline{CL}$ are perpendicularly to each other i.e. $m\angle CLB = 90^\circ$

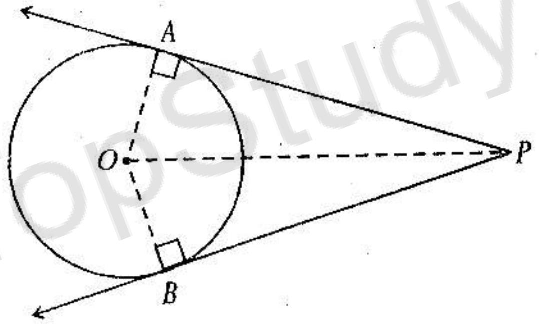
Construction: Take any point M on $\overleftrightarrow{AB}$. Join C and M by line segment $\overline{CM}$.

Proof:

Statements	Reasons
Since M lies outside the circle	Construction
$\therefore \ m\overline{CM} > m\overline{CL}$	For any point lying in the exterior of a circle, its distance from centre of the circle is always greater than the radius.
As L lies on the circle $\overline{CL}$ is the shortest of all segments drawn from C on $\overleftrightarrow{AB}$	
i.e. $\overline{CL} \perp \overleftrightarrow{AB}$	
or $m\angle CLB = 90^\circ$	

Theorem 6

The two tangents drawn to a circle from a point outside it, are equal in length.



Given:

A circle with centre at O and the tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ to the circle from a point P outside the circle.

To Prove:

$m\overline{PA} = m\overline{PB}$

Construction:

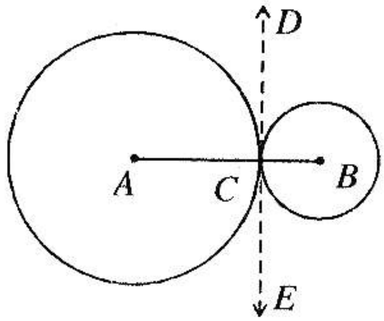
Join O to P, A and B

Proof:

Statements	Reasons
In $\Delta OAP \leftrightarrow \Delta OBP$	
$\overline{OA} \cong \overline{OB}$	Radii of same circle
$\overline{OP} \cong \overline{OP}$	Common
$m\angle OAP = m\angle OBP$ $= 90^\circ$	Radial segment and tangent line are perpendicularly to each other
$\therefore \ \Delta OAP \cong \Delta OBP$	H.S $\cong$ H.S
$m\overline{PA} = m\overline{PB}$	Corresponding sides of congruent triangles.

Theorem 7

If two circles touch externally, the distance between their centres is equal to the sum of their radii.



Given:

Two circles with centres at A and B touching externally at point C.

i.e., m $\overline{AC}$, m $\overline{BC}$ are the radii of the two circles.

To Prove:

$$m\overline{AB} = m\overline{AC} + m\overline{CB}$$

Construction:

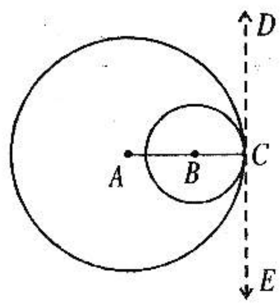
Draw a common tangent line $\overleftrightarrow{DE}$ at the point C where circles are touching externally.

Proof:

Statements	Reasons
In circle centre at A, $\overleftrightarrow{DE}$ is a tangent line at C, and $\overline{AC}$ is the radial segment	
$\therefore$ m$\angle$DCA = 90° (i)	A radial segment is perpendicularly to a tangent line.
Similarly, m$\angle$DCB = 90° (ii)	
m$\angle$DCA + m$\angle$DCB = 180° or m$\angle$ACB = 180° $\therefore$ Points A, C and B are collinear.	By adding (i) and (ii) $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays of a straight angle.
i.e. m $\overline{AB}$ = m$\overline{AC}$ + m$\overline{CB}$	C lies in between A and B

Theorem 8

If two circles touch internally, the distance between their centres is equal to the difference their radii of.



Given: Two circles with centers at A and B touch internally at point C .i.e. m$\overline{AC}$ and m $\overline{BC}$ are the radii of the two circles such that m$\overline{AC}$ > m$\overline{BC}$

To Prove:

m$\overline{AB}$ = m$\overline{AC}$ - m$\overline{BC}$

Construction:

Draw a common tangent line $\overleftrightarrow{DE}$ at the point C where circles are touching internally.

Proof:

Statements	Reasons
In circle with centre at A, $\overleftrightarrow{DE}$ is a tangent line at C, and $\overline{AC}$ is the radial segment	Construction
$\therefore$ m$\angle$DCA = 90° (i)	A radial segment is perpendicularly to a tangent line
Similarly, m$\angle$DCB = 90° (ii)	
m$\angle$DCA = m$\angle$DCB = 90°	From (i) and (ii)
$\therefore$ m$\overline{AB}$ + m$\overline{BC}$ = m$\overline{AC}$	Because $\overline{CB}$ and $\overline{CA}$ are the common arms of the same right angle $\angle$DCB.
Therefore, $m\overline{AB} = m\overline{AC} - m\overline{BC}$	

Corollary 1: If two congruent circles touch each other externally, the distance between their centres is equal to their diameter.

Corollary 2: If two congruent circles touch each other internally, the distance between their centres is equal to zero.

EXAMPLES

Q.1 Fill in the Blanks:

(i) A set of all points of a plane equidistant from a fixed point is called a __________.

(ii) The distance of any point of a circle from its centre is called _____.

(iii) A line segment whose two end points are common with a circle is called ________.

(iv) A chord through the centre of the circle is called ______.

(v) Two circles are congruent if their ________ are congruent.

Ans. (i) Circle (ii) Radius (ii) Chord (iv) Diameter (Central Chord) (v) Radii

Q.2 Write true or false

(i) Diameter contains three points of a circle.

(ii) A chord is a line segment passing through the two points of a circle.

(iii) The set of all points lying inside a circle is called the circular region.

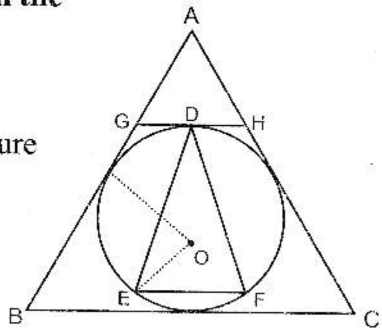
(iv) A line containing two points of a circle is called a secant line.

(v) The angle subtended by any arc of a circle at any point lying in the interior of a circle is called a central angle.

Answers: (i) False (ii) True (iii) False (iv)True (v) False

Q.3 Write names of the circles associated with the

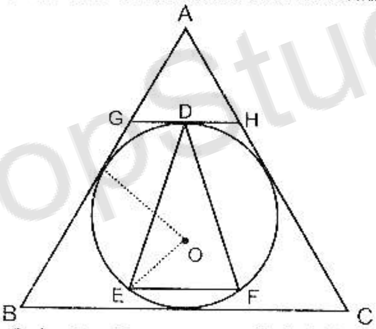
i) Δ ABC
ii) Δ DEF
iii) Δ AGH
as shown in figure



Sol:

i) In-circle (ii) Circum Circle
ii) Escribed Circle

Q.4 Study the figure as shown and indicate which of the following statements are true and which false?



i. O is the in-centre of ΔABC
Sol: True
ii. Circle at centre O is escribed circle of ΔDEF
Sol: False
iii. Angle bisectors of Δ ABC are meeting at O.
Sol: True
iv. Line segment GH is tangent to the circle
Sol: True
v. Perpendicular bisectors of ΔDEF are meeting at O.
Sol: True
vi. Angle bisector of < HGB will pass through O.
Sol: True
vii. Angle bisector of $\angle$GAH, $\angle$ HGB and $\angle$GHC will meet at O.
Sol: True
viii. Area of circle is greater than area of ΔABC.
Sol: False
ix. Area of ΔDEF is less than area of circle
Sol: True
x. $\overline{OE}$ is called in-radius with respect to Δ DEF
Sol: True

Q.5 Fill in the blanks:

(i) A diameter of a circle perpendicular to a chord always_____ the chord.
(ii) The diameter bisecting a chord is __________ to the chord.
(iii) Two congruent chords of a circle are ______ from the centre of the circle.
(iv) If two chords of a circle are equidistant from the centre of a circle they will be _________.
(v) The line which is at a distance, equal to radius of the circle from the centre of a circle will be ______ to the circle.

Sol: (i) Bisects
(ii) Perpen icular
(iii) Equiustant
(iv) Congruent
(v) Tangent

Q.6 Fill in the blanks by correct answer.

(i) Through three non collinear points _____ circle can pass.

(ii) Centre of the circle lies on ____ line segments joining three non collinear points on the circle.

(iii) If a circle passes through three non collinear points then the distance of any point lying on the circle from its _____ is same.

(iv) The central angle of minor arc is ______ than the inscribed angle of major arc.

(v) All angles inscribed in a major arc are______.

Sol: (i) Only one

(ii) Point of intersection or perpendicular bisectors

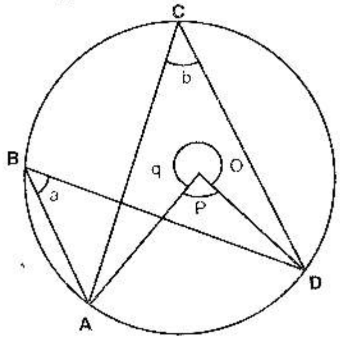
(iii) Centre (iv) Double

(v) Acute

Q.7 In fig, there is a circle centre at O.

If $m\angle P = 100^0$

Find $m\angle a, m\angle b$ and $m\angle q$



Note: The measure of a central angle of a minor arc of a circle is double in measure of inscribed angle of the corresponding major arc.

Sol:

Consider a circle with centre at O.

Also $m\angle P = 100^0$

$m\angle a$, $m\angle b$ & $m\angle q$ are to be determined

Here $m\angle P = 100^0$

So $m\angle b = \frac{1}{2} m\angle p$

$= \frac{1}{2}(100^0)$

$m\angle b = 50^0$

As $m\angle P + m\angle q = 360^0$

$100^0 + m\angle q = 360^0$

$\Rightarrow m\angle q = 260^0$

As all angles inscribed in a major arc are equal in measure

So $m\angle a = m\angle b$

But $m\angle b = 50^0$

So $m\angle a = 50^0$

Q.8 Fill in the blanks:

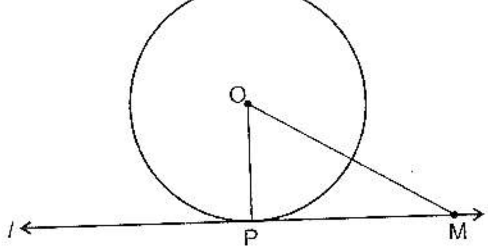
(i) A line which is perpendicular to a radial segment at its outer end is ________ to the circle.

(ii) If a line is a tangent to a circle at a point it will be ______ to the radial segment.

(iii) The shortest distance between a tangent line from centre of a circle is equal to ________.

(iv) The perpendicular at the point of contact on a tangent line will pass through _______ of circle.

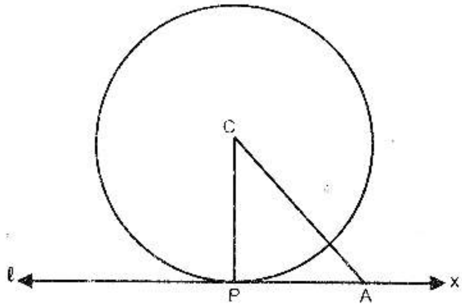
(v) In the figure, l is a tangent line to a circle, at P, then $m\overline{OE}$ is always _______ than $m\overline{OP}$



Sol:

(i) Tangent (ii) Perpendicular
(iii) Radius (iv) Center
(v) Greater

Q.9 In fig. l is a tangent line to a circle with centre at C. Given that
$m\angle PCA = 30^0$



Find $m\angle CAP$

Sol: Given a circle with centre at C as l is a tangent line to circle.

So $l \perp \overline{CP}$

So $m\angle APC = 90^0$

Also given $m\angle PCA = 30^0$

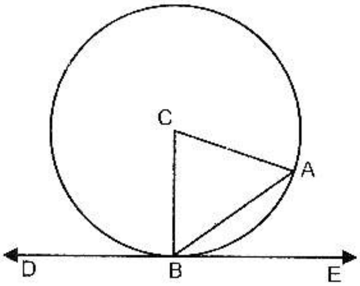
$m\angle P + m\angle PCA + m\angle CAP = 180^0$

$90^0 + 30^0 + m\angle CAP = 180^0$

So $m\angle CAP = 180^0 - 120^0$

$= 60^0$

Q.10 A circle centre at C touches $\overleftrightarrow{DE}$ at B as shown in fig. Find $m\angle CBA$, $m\angle BAC$ and $m\angle ACB$ if $m\angle ABE = 40^0$



Sol: Given a circle centre at C touches $\overleftrightarrow{DE}$ at B. Also $m\angle ABE = 40^0$

To find $m\angle CBA$, $m\angle BAC$ and $m\angle ACB$

Proof:

As circle touches $\overleftrightarrow{DE}$

So $\overleftrightarrow{DE} \perp \overline{BC}$

Hence $m\angle CBE = 90^0$

Also $m\angle ABE = 40^0$

So $m\angle CBA = 90^0 - 40^0$

$= 50^0$

As $\overline{AC} \cong \overline{BC}$ (Radii of same circles)

So $m\angle CBA = m\angle BAC$

So $m\angle BAC = 50^0$

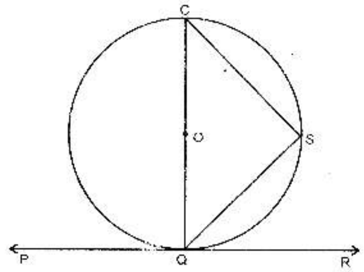
$M\angle ACB + m\angle CBA + m\angle BAC = 180$

Now $m\angle ACB = 180^0 - (50^0 + 50^0)$

$= 180^0 - 100^0$

$= 80^0$

Q.11 In fig $\overline{PR}$ is a tangent to the circle with centre O. If



$m\angle PQS = 150^0$. Find $m\angle QCS$

Sol: Given $\overline{PR}$ is a tangent to circle with centre O. Also $m\angle PQS = 150^0$

To find $m\angle QCS$

Proof: As $\overline{PR}$ is tangent to circle with centre at O

So $\overline{PR} \perp \overline{CQ}$

$\therefore \quad m\angle OQR = 90^0$

As $m\angle PQS + m\angle SQR = 180^0$

So $m\angle SQR = 180^0 - m\angle PQS$

$= 180^0 - 150^0$

$= 30^0$

But $\quad m\angle OQR = 90^0$

So $\quad m\angle OQS = 90^0 - 30^0 = 60^0$

Also as $m\angle CSQ = 90^0$ (Angle inscribed in semi circle)

So $m\angle QCS = 90^0 - 60^0$

$= 30^0$

Q.12 Fill in the blanks with appropriate answers.

(i) Two tangents drawn to a circle from a point outside are ______ in length.

(ii) If a tangent is drawn from a point outside the circle of radius 3cm is of length 5cm, then length of line segment joining the point and the centre is __________.

(iii) If two circles of radii 6cm and 2cm touch externally, then the distance between their centres is equal to __________.

(iv) If two circles of radii 8 cm and 4cm touch internally, then distance between their centres is equal to __________.

(v) If two circles touch each other externally, then their point of contact always lies on________.

SOL:

(i) Equal (ii) $\sqrt{34}cm$ (iii) 8cm

(iv) 4 cm (v) Line segment joining centres of circles

Q.13 Fill in the blanks with appropriate answers

(i) The set of all points of a plane which are equidistant from a fixed point is called a __________.

(ii) A diameter is a chord which passes through the ______ of the circle.

(iii) Central angle is an angle subtended by an arc at the ______ of the circle.

(iv) Two circles are ________ if their radii are congruent.

(v) The measure of an angle inscribed in a semi circle is equal to ______.

Sol: (i) Circle (ii) Centre (iii) Centre (iv) Congruent (v) 90^0

Q.14 Classify each statement as true T or False F.

(i) Every diameter of a circle is also a chord of the circle.

(ii) A circle has exactly two radii.

(iii) A chord of a circle is also a segment of a tangent line to the circle.

(iv) If a diameter is perpendicular to a chord, then it bisects the chord.

(v) An inscribed angle is an angle whose vertex lies at the centre of the circle.

Sol: (i) True (ii) False (iii) False (iv) True (v) False

Q.15 Select the correct answer out of a, b, c, d and write in the blanks in front of each question.

(i) $\overline{AB}$ is called a

(a) Chord (b) Tangent

(c) Diameter (d) Secant

Ans: Chord

(ii) $\overleftrightarrow{EF}$ is called a

(a) Chord (b) Tangent

(c) Diameter (d) Secant

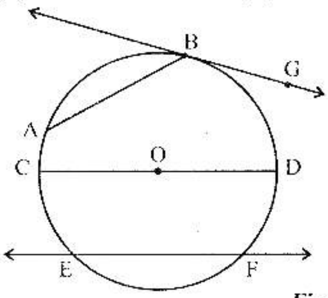


Fig (i)

Ans: Secant

(iii) $\overline{BG}$ is called a

(a) Chord (b) Tangent

(c) Diameter (d) Secant

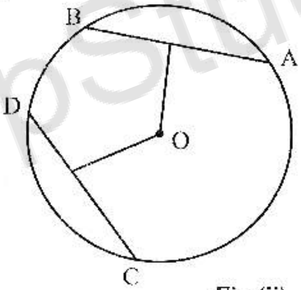


Fig (ii)

Ans: Tangent

(iv) $\overline{CD}$ is called a

(a) Chord (b) Tangent

(c) Diameter (d) Secant

Ans: Diameter

(v) Refer to fig (ii) chords $\overline{AB}$ *and* $\overline{CD}$ of the circle are equidistant from its centre O are.

(a) Congruent (b) Not Congruent

(c) Parallel (d) Perpendicular

Ans: Congruent

Q.16 For parts (i) to (v) see fig on the right and match entries in column I with the correct answer in column II.

Sol:

The matched entries in column I and II are as:

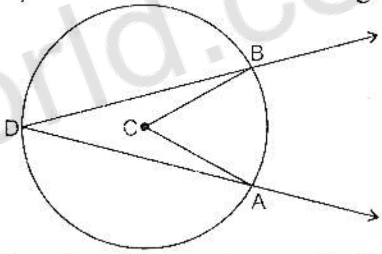
i) $\widehat{AB}$ Minor arc

ii) $\angle ACB$ Central angle

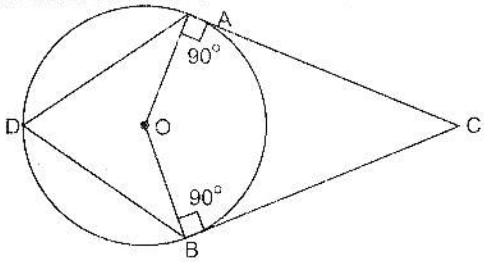
iii) $\widehat{ADB}$ Major arc

iv) $\overline{CB}$ Radial Segment

v) $\angle$ADB Inscribed angle



Q.17 In fig. There is a circle with centre at O. Show that $\angle$C and $\angle$AOB are supplementary If m $\angle C = 40^0$ then find m$\angle$AOB and m$\angle$ADB.



SOL: Given a circle with centre at O. TO show $\angle$C and $\angle$AOB are supplementary.

As in quadrilateral ABCD, $m\angle B = 90^0$ and $m\angle A = 90^0$

So $m\angle A + m\angle B = 180^0$

But as $m\angle A + m\angle AOB + m\angle B + m\angle C = 360^0$

So $m\angle AOB + m\angle C = 180^0$

Hence $\angle$C and $\angle$AOB are supplementary

Now as given $m\angle C = 40^0$

So $m\angle AOB = 180^0 - 40^0$

$= 140^0$

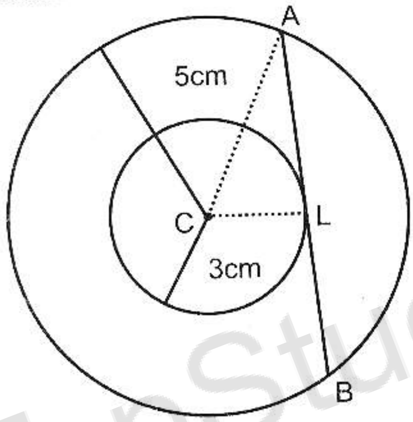
Also as $m\angle ADB = \frac{1}{2}\, m\angle AOB$

$= \frac{1}{2}(140^0)$

$= 70^0$

So $m\angle ADB = 70^0$

Q.18 In fig two concentric circles with common centre C and of radii of measures of 5cm and 3cm. Chord $\overline{AB}$ touches externally the smaller circle. Find $m\overline{AB}$



Sol:

Given two concentric circles with centre C and chord $\overline{AB}$ touches the small circle externally.

To find $m\overline{AB}$

Construction: Join C with L and A

As $\overline{AB}$ touches the small circle

So $m\angle ALC = 90^0$

Applying Pythagoras theorem as Δ ALC

$$(\overline{AC})^2 = (\overline{AL})^2 + (\overline{LC})^2$$

$$(5)^2 = (\overline{AL}) + (3)^2$$

$\therefore$ $\overline{CL}$ is radius of small circle and $\overline{AC}$ is radius of big circle

So

$$25 = (\overline{AL})^2 + 9$$

$$(\overline{AL})^2 = 25 - 9$$

$$(\overline{AL})^2 = 16$$

So $m\overline{AL} = 4$ cm

Hence $m\overline{AB} = 2m\overline{AL}$

$= 2(4)$

$= 8$ cm

OBJECTIVE

Q. 1. Four answers of each item are given from which only one is true. Tick the correct answer.

1. The set of all points in a plane which are equidistant from a fixed point of the plane is called.
   (a) Centre (b) Circle
   (c) Circumference (d) diameter
2. In circle the fixed point is called.
   (a) centre (b) radius
   (c) diameter
   (d) circumference
3. The distance between any point on the circle and its centre is called _____ of the circle.
   (a) radius (b) centre
   (c) diameter
   (d) circumference
4. All radial segments of a circle are ______ in length.
   (a) equal (b) not equal
   (c) parallel
   (d) perpendicular
5. The length of line joining all points of the circle is called_______.
   (a) circumference (b) centre
   (c) chord (d) radius
6. A line segment whose end points are any two points of a circle is called ______of the circle.
   (a) chord (b) diameter
   (c) radius (d) radical segment
7. A chord passing through the centre of the circle is called a _______ of the circle.
   (a) chord (b) diameter
   (c) radius (d) radical segment
8. Any portion or part of a circle is called ______ of the circle.
   (a) arc (b) radius
   (c) half circle (d) major arc
9. The portion of a circle intercepted by any central chord is a______.
   (a) circle (b) half circle
   (c) arc (d) major arc
10. An arc which is included in a semi circle is called______.
    (a) major arc (b) minor arc
    (c) arc (d) semi arc
11. An arc which includes a semi circle is called______.
    (a) major arc (b) minor arc
    (c) arc (d) circumference
12. Minor arc is always ______ the semi-circle.
    (a) less than (b) greater than
    (c) equal to (d) congruent to
13. Major arc is always ______ the semi-circle.
    (a) less than (b) greater than
    (c) equal to (d) congruent to
14. The angle subtended by an arc at the centre of a circle is called______.
    (a) central angle
    (b) inscribed angle
    (c) angle (d) radius
15. Two circles are congruent if their ________ are equal.
    (a) radii (b) chords
    (c) centre (d) arcs

16. A chord of a circle divides the circular region into two parts these regions are called______ of the circle.
(a) sectors (b) radius
(c) segments (d) half circle

17. The region bounded by the chord and the minor arc is called_______.
(a) major segment
(b) minor segment
(c) segment (d) circular region

18. The region bounded by the chord and the major arc is called_________.
(a) major segment
(b) minor segment
(c) segment (d) circular region

19. Minor segment of a circle is always __________ semi circular region of the circle.
(a) less than (b) greater than
(c) equal to (d) congruent to

20. Major segment of a circle is always __________ the semi circular region of the circle.
(a) greater than (b) less than
(c) equal to (d) included in

21. The circular region bounded by an arc of a circle and its two corresponding radial segments is called_________ of circle.
(a) major segment (b) sector
(c) radius
(d) minor segment

22. A line which has two distinct points in common with a circle is called a ______.
(a) secant line (b) tangent line
(c) radial segment (d) chord

23. A line which intersects a circle at one point only and none of its points lie in the interior of the circle is called _________ to the circle.
(a) tangent line (b) secant line
(c) radius (d) chord

24. Those circles which has only one point in common are called_________.
(a) tangent line
(b) tangent circles
(c) concentric circles
(d) congruent circles

25. If a tangent is drawn from a point outside a circle to the circle then the distance between that point and point of tangency is called_________ segment.
(a) tangents
(b) length of tangent
(c) direct common tangent
(d) tangential segment

26. Circles having a common centre are called___________.
(a) tangent circles
(b) concentric circles
(c) equal circles (d) common circles

27. The circle which passes through the three vertices of a triangle is called_____ of the triangle.
(a) circum-circle (b) inscribed circle
(c) escribed circle (d) tangent circles

28. The circle which touches the three sides of a triangle internally is called ________ of the triangle.
(a) circum circle (b) in-circle
(c) e-circle (d)tangent circle

29. The circle which touches one side externally and the other two produced sides of a triangle internally is called________.
(a) escribed circle
(b) inscribed circle
(c) circum circle (d) in-circle

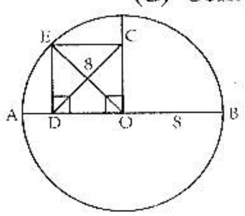
30. If a diameter of a circle is perpendicular to a chord, it ________ the chord.
(a) intersects (b) bisects
(c) is perpendicular to (d) is parallel to

31. If a diameter of circle bisects a chord it will be _______ to the chord.
(a) perpendicular (b) congruent
(c) parallel (d) anti-parallel

32. If two chords of a circle are _____ then they will be equidistant from the centre.
(a) congruent (b) parallel
(c) bisecting
(d) perpendicular

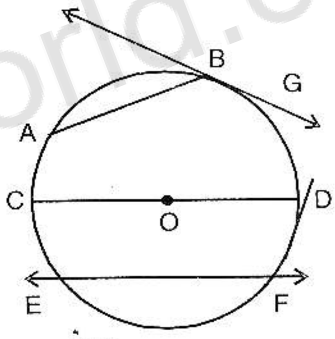
33. In fig. $\overline{AB}$ is a diameter of the circle with centre O and OCED is a rectangle

if $m\overline{DB}$ = 14cm and $m\overline{AD}$ = 2cm then length of mDC
(a) 4 cm (b) 10cm
(c) 5cm (d) 8cm



Note: Since AB is a diameter
So $m\overline{DC} = m\overline{ED}$ (diagonal)

$m\overline{AB} = m\overline{AD} + m\overline{BD}$
$m\overline{AB} = 2 + 14$
$m\overline{AB} = 16\text{cm}$
$m\overline{OA} = m\frac{AB}{2} = \frac{16}{2} = 8\text{cm}$
$m\overline{OA} =$ 8 cm -------------- (1)
$m\overline{OE} = m\overline{OA} = 8$ cm
$m\overline{DE} = m\overline{OE}$
$m\overline{DE} = 8\text{cm}$



34. In figure, $\overline{AB}$ is called a _____.
(a) chord (b) tangent
(c) diameter (d) secant

35. $\overleftrightarrow{EF}$ is called ___________.
(a) chord (b) tangent
(c) diameter (d) secant

36. $\overline{CD}$ is Called ___________.
(a) tangent (b) chord
(c) diameter (d) secant

37. $\overrightarrow{BG}$ is called
(a) chord (b) tangent
(c) diameter (d) secant

38. If two circles of radii 6cm and 2cm touch externally the distance between their centres is equal to ________.
(a) 8cm (b) 4cm
(c) 2cm (d) 2cm

39. If two circles of radii 8cm and 4cm touch internally then distance between their centres is equal to ______.

(a) 2cm (b) 4cm
(c) 8cm (d) 12cm

40. Angle inscribed in a major arc is ________ angle.
(a) acute (b) obtuse
(c) right (d) central

41. The measure of a central angle of a minor arc of circle is _________ in measure of inscribed angle of corresponding major arc
(a) double (b) triple
(c) half (d) less

42. Two circles touch externally the distance between their centres is _______ the sum of their radii.
(a) equal to (b) half of
(c) double (d) greater than

43. Two circle touch internally, the distance between their centres is equal to the ______of their radii
(a) division (b) difference
(c) addition
(d) multiplication

44. If two congruent circles touch each other externally the distance between their centres is equal to __________.
(a) their radii (b) their diameter
(c) zero (d) their chord

45. If two congruent circles touch each other internally the distance between their centres is equal to __________.
(a) their diameter (b) zero
(c) their radii
(d) their circumstance

46. If two chords of a circle are equidistant from the centre of a circle they will be ______.
(a) congruent (b) parallel
(c) concurrent (d) anti-parallel

47. The line which is at a distance, equal to radius of the circle from the centre of a circle will be ____ to the circle.
(a) tangent (b) secant
(c) perpendicular (d) parallel

48. Through three non collinear points ____ circle can pass.
(a) only one (b) two
(c) three (d) infinite

49. If a circle passes through three non collinear points then the distance of any point lying on the circle from its____ is same.
(a) centre (b) diameter
(c) chord
(d) circumference

50. A line which is perpendicular to a radial segment of a circle at its outer end on the circle is _____ to the circle.
(a) tangent (b) secant
(c) equal (d) parallel

51. If a line is a tangent to a circle at a point it will be ______ to the radial segment.
(a) parallel (b) perpendicular
(c) equal (d) congruent

52. The shortest distance between a tangent line from centre of a circle is equal to ____.
(a) radius (b) chord
(c) diameter (d) centre

53. The perpendicular at the point of contact on a tangent line will pass through ____ of circle.

(a) centre (b) radius
(c) diameter (d) chord

54. Two tangents drawn to a circle from a point outside are _____ in length.
(a) equal (b) greater
(c) smaller (d) double

55. If a tangent is drawn from a point outside the circle of radius 3cm is of length 5cm, then length of line segment joining the point and the centre is _________.
(a) $\sqrt{34}$ cm (b) $\sqrt{32}$ cm
(c) $\sqrt{30}$ cm (d) $\sqrt{35}$ cm

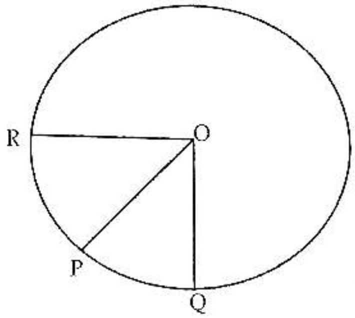
56. It two circles touch each other externally then their point of contact always lies on _______.
(a) Line segment joining their centres
(b) radial segment
(c) chord (d) diameter

57. Central angle is an angle subtended by an are at the _____ of the circle.
(a) centre (b) radius
(c) chord (d) circumference

58. Two circles are _____ if their radii are congruent.
(a) congruent (b) tangents
(c) different (d) concentric

59. The measure of an angle inscribed in a semi circle is equal to ___.
(a) 90^0 (b) 80^0
(c) 45^0 (d) 60^0

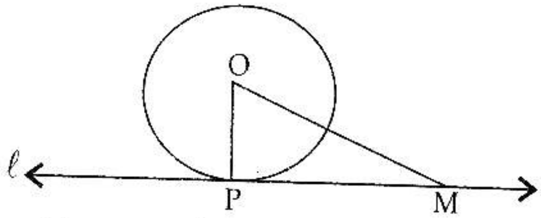
60. In figure region OPQ and OPR are _______ of the circle.



(a) sectors (b) radius
(c) chord (d) minor segment

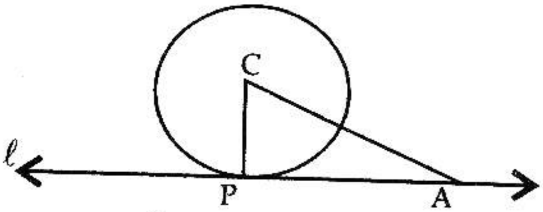
61. All angles inscribed in a major are _____ in measure.
(a) equal (b) unequal
(c) congruent (d) obtuse

62. In figure, ℓ is a tangent line to a circle at P, then mOM is always _______ mOP.



(a) greater than (b) smaller than
(c) equal to (d) perpendicular to

63. In figure, ℓ is a tangent line to a circle centre at C. Given that m<PCA = 30^0, then m$\angle$CAP = _____.



(a) 60^0 (b) 30^0
(d) 50^0 (d) 90^0

Answers

1.	b	**2.**	a	**3.**	a	**4.**	a	**5.**	a	**6.**	a	**7.**	b
8.	a	**9.**	b	**10.**	b	**11.**	a	**12.**	a	**13.**	b	**14.**	a
15.	a	**16.**	c	**17.**	b	**18.**	a	**19.**	a	**20.**	a	**21.**	b
22.	a	**23.**	a	**24.**	b	**25.**	b	**26.**	b	**27.**	a	**28.**	b
29.	a	**30.**	b	**31.**	a	**32.**	a	**33.**	d	**34.**	a	**35.**	d
36.	c	**37.**	b	**38.**	a	**39.**	b	**40.**	a	**41.**	a	**42.**	a
43.	b	**44.**	b	**45.**	b	**46.**	a	**47.**	a	**48.**	a	**49.**	a
50.	a	**51.**	b	**52.**	a	**53.**	a	**54.**	a	**55.**	a	**56.**	a
57.	a	**58.**	a	**59.**	a	**60.**	a	**61.**	a	**62.**	a	**63.**	a